Aryabhatta Knowledge University (AKU)

EE

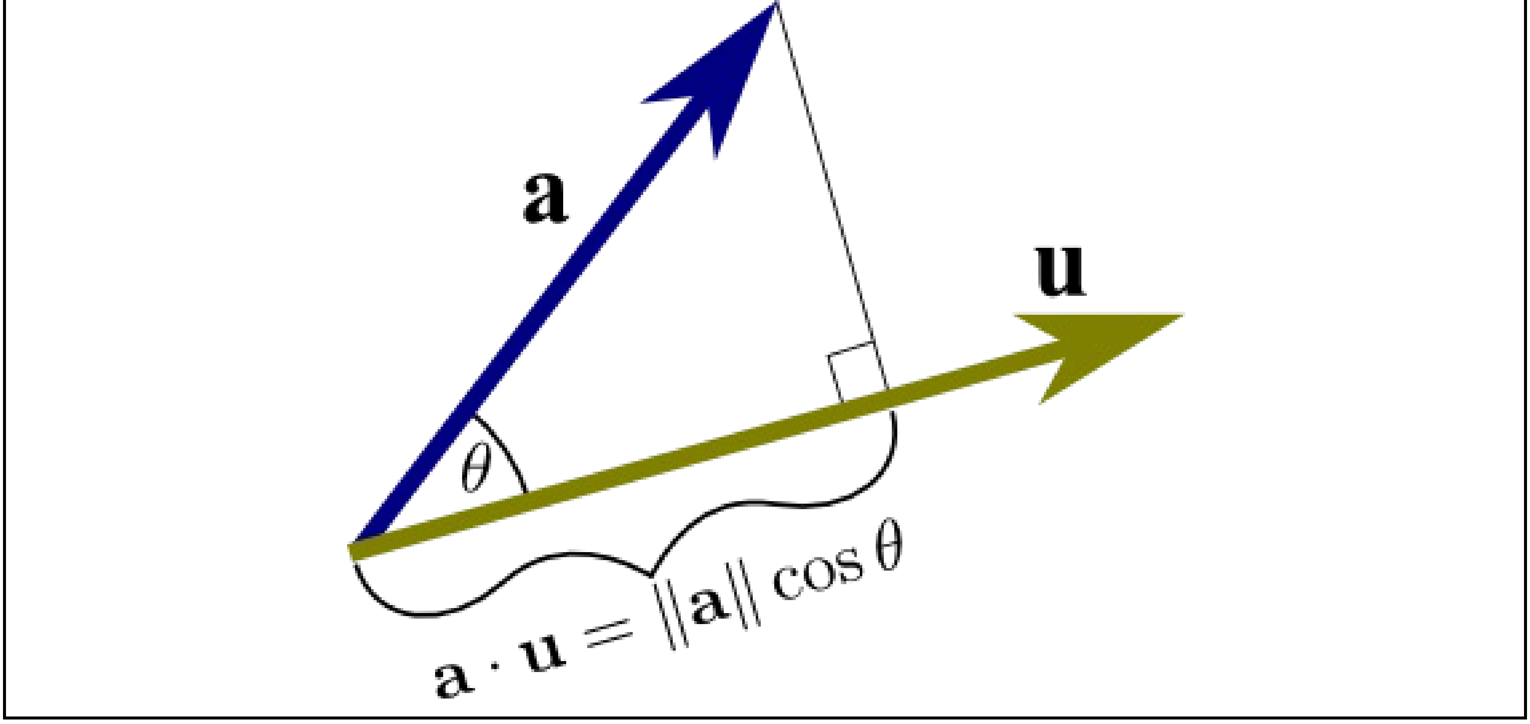
Electromagnetic fields

Solved Exam Paper 2019

Question: What is dot product? Explain its significance and applications.

Answer: The dot product of aa with unit vector uu, denoted a·u, is defined to be the projection of aa in the direction of uu, or the amount that aa is pointing in the same direction as unit vector uu. Let's assume for a moment that aa and uu are pointing in similar directions. Then, you can imagine a·uca·u as the length of the shadow of aa onto uu if their tails were together and the sun was shining from a direction perpendicular to uu. By forming a right triangle with aa and this shadow, you can use geometry to calculate that

$a \cdot u = \|a\| \cos\theta$



Significance: It tells you about how much of the vectors are in the same direction

1. Application: Finding angle between 2 vectors or 2 straight lines, angle between 2 intersecting planes, angle between plane and straight line.
2. Finding projection of a vector onto another unit vector. Applications include finding projection of a force onto a specified axis.
3. Checking whether 2 vectors are perpendicular.
4. Finding work done by a force.
5. Multiplication of matrices in linear algebra involve taking dot product of the row in left matrix with column in right matrix

Question: Discuss the following terms as applied to vectors fields:

I) Gradient

II) Divergence

III) Curl and its physical interpretation

Answer:

I)Gradient: For a real-valued function f(x,y,z)f(x,y,z) on R3, the gradient ∇f(x,y,z)∇f(x,y,z) is a vector-valued function of R3, that is, its value at a point (x,y,z)(x,y,z) is the vector

$$\nabla f(x,y,z)=(\partial f/\partial x,\partial f/\partial y,\partial f/\partial z)$$

$$=\partial f/\partial x\ i+\partial f/\partial y\ j+\partial f/\partial zk$$

$$\nabla f(x,y,z)=(\partial f/\partial x,\partial f/\partial y,\partial f/\partial z)$$

$$=\partial f/\partial x\ i+\partial f/\partial y\ j+\partial/f\partial z\ k$$

In R3, where each of the partial derivatives is evaluated at the point (x,y,z)(x,y,z). So in this way, you can think of the *symbol* ∇ as being "applied" to a real-valued function ff to produce a vector ∇f.

It turns out that the divergence and curl can also be expressed in terms of the symbol ∇. This is done by thinking of ∇ as a *vector* in R3, namely

$$\nabla=\partial/\partial x\ i+\partial/\partial y\ j+\partial/\partial z\ k.$$

II) Divergence

it is often convenient to write the divergence div **f** as ∇·f, since for a vector field

$$f(x,y,z)=f1(x,y,z)i+f2(x,y,z)j+f3(x,y,z)kf(x,y,z)$$

$$=f1(x,y,z)i+f2(x,y,z)j+f3(x,y,z)k,$$

The dot product of **f** with ∇ (thought of as a vector) makes sense:

$$\nabla\cdot f=(\partial/\partial xi+\partial/\partial yj+\partial/\partial zk)\cdot(f1(x,y,z)i+f2(x,y,z)j+f3(x,y,z)k)$$

$=(\partial/\partial x)(f1)+(\partial/\partial y)(f2)+(\partial/\partial z)(f3)$

$=\partial f1/\partial x+\partial f2/\partial y+\partial f3/\partial z=$

$=\text{div } f$

III) Curl and its physical interpretation: The curl of a vector field, denoted $\text{curl}\,(\mathbf{F})$ or $\nabla\times\mathbf{F}$ (the notation used in this work), is defined as the vector field having magnitude equal to the maximum "circulation" at each point and to be oriented perpendicularly to this plane of circulation for each point. More precisely, the magnitude of $\nabla\times\mathbf{F}$ is the limiting value of circulation per unit area. Written explicitly,

Physical Interpretation:

The **curl** of a vector field measures the tendency for the vector field to swirl around. Imagine that the vector field represents the velocity vectors of water in a lake. If the vector field swirls around, then when we stick a paddle wheel into the water, it will tend to spin.

Question: What is the various types of charge distributions?

Answer:

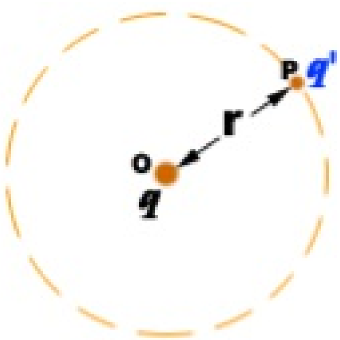
Type of charge distribution	**Denoted by**	**Un**
Line **Charge**	λ (Line **charge density**)	C/
Surface **Charge**	σ (surface **charge density**)	C/
Volume **Charge**	ρ (volume **charge density**)	C/

Question: State the units of electric field intensity E and

explain the method of obtaining E at a point in Cartesian system, due to point charge Q.

Answer: Electric Field Intensity (E) = $q/[4\pi\varepsilon d^2]$ NC^{-1}

Consider a point charge q called SOURCE CHARGE placed at a point in space. To find its intensity at a point ‘p’ at a distance ‘r’ from the po charge we place a test charge 'q'.



The force experienced by the test charge q’ will be:

F = Eq’----(1)

According to coulomb's law the electrostatic force between then given by:

$$F = \frac{Kqq'}{r^2}$$

We know that:

$$E = \frac{F}{q'}$$

Or

$$E = \frac{1}{q'} \times F$$

Putting the value of 'F' we get : www.citycollegiate.com

$$E = \frac{1}{\cancel{q'}} \times \frac{Kq\cancel{q'}}{r^2}$$

$$E = \frac{Kq}{r^2}$$

But $K = \frac{1}{4\pi \varepsilon_o}$

$\therefore$

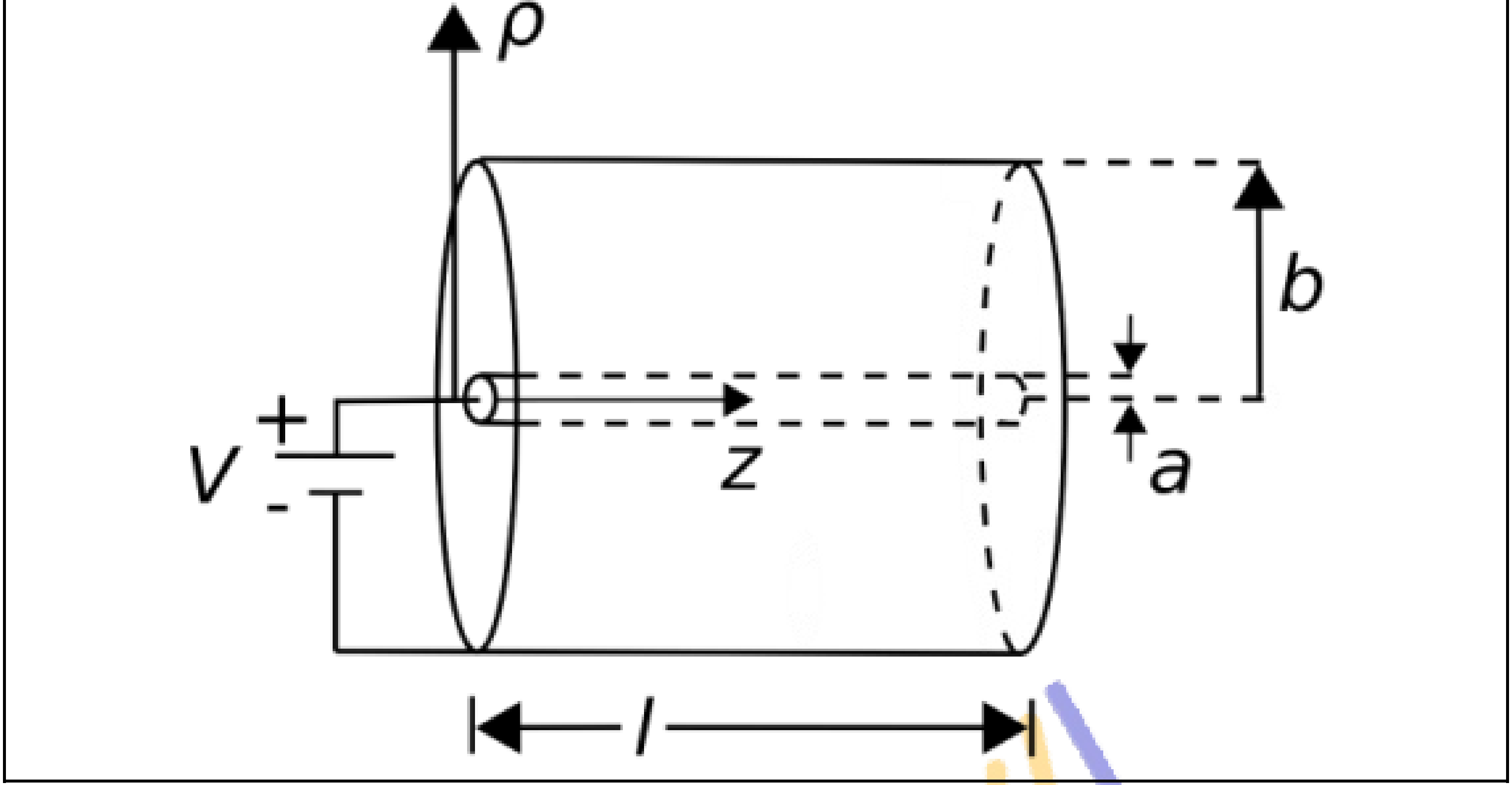
$$\boxed{E = \frac{1}{4\pi \varepsilon_o} \times \frac{q}{r^2}}$$

This shows that the electric intensity due to a point charge is dire proportional to the magnitude of charge q and inversely proportiona the square of distance

Question: Derive an expression for the capacitance per unit length of a coaxial cable with permittivity e, inner diameter d and outer diameter D.

Answer: Let us now determine the capacitance of coaxially-arranged conductors, shown in.

The structure as consisting of two concentric perfectly-conducting cylinders of radii aa and bb, separated by an ideal dielectric having permittivity ϵ We place the +z+z axis along the common axis of the concentric cylinders so that the cylinders may be described as constant-coordinate surfaces ρ=d and ρ=D



In this section, we shall find the capacitance by assuming a total charge Q+Q+ on the inner conductor and integrating over the associated electric field to obtain the voltage between the conductors. Then, capacitance is computed as the ratio of the assumed charge to the resulting potential difference

The first step is to find the electric field inside the structure. This is relatively simple if we assume that the structure has infinite length (i.e., l→∞l→∞), since then there are no fringing fields and the internal field will be utterly constant with respect to zz. In the central region of a finite-length capacitor, however, the field is not much different from the field that exists in the case of infinite length, and if the energy storage in fringing fields is negligible compared to the energy storage in this central region then there is no harm in assuming the internal field is constant with zz. Alternatively, we may think of the length ll as pertaining to one short section of a much longer structure and thereby obtain the capacitance *per length* as opposed to the total capacitance.

To determine the capacitance,

$C \triangleq Q+/V$

Where $Q+Q+$ is the charge on the positively-charged conductor and VV is the potential measured from the negative conductor to the positive conductor. The charge on the inner conductor is uniformly-distributed with density

$\rho l = Q+/\text{length}$

Which has units of C/m. Now we will determine the electric field intensity EE, integrate EE over a path between conductors to get VV, =

“Electric Field Due to an Infinite Line Charge using Gauss’ Law,” where we found

$E = \rho \hat{\ } \rho l 2\pi \epsilon s \rho$

This is a consequence of Gauss’ Law

$\oint S D \cdot ds = Qencl$

$V = -\int C E \cdot dl$

)

Wrapping up:

$C \triangleq Q+/V = \rho l l(\rho l/2\pi \epsilon s)\ln(D/d)$

Note that factors of $\rho l \rho l$ in the numerator and denominator cancel out, leaving:

$C = 2\pi \epsilon s l\ /\ (\ln(b/a))$

Question: Discuss the Laplace’s and Poisson’s equations

Answer:

Laplace's equation is a linear, homogeneous, partial differential equation. It has the form:

$\nabla^2 u=0$

or if $u\in Rn$ then:

$\partial^2 u/\partial x1^2+.........+\partial^2 u/\partial xn^2=0$

Poisson's equation is simply the inhomogeneous version of Laplace's equation. That means it is of the form:

$\partial^2 u/\partial x1^2+.........+\partial^2 u/\partial xn^2=f(x1,...,xn)$

for $u\in Rn$

$f\in C(Rn)$

As a final note I am not sure about whether there is an strict, universal definition on the source function ff. I have gone with the one I have seen used most often but I am interested if anyone knows whether there are ever more relaxed requirements for f.

Question: Find H at the centre of the square loop of side L in xy plane at the origin as centre, carrying current I.

Answer:

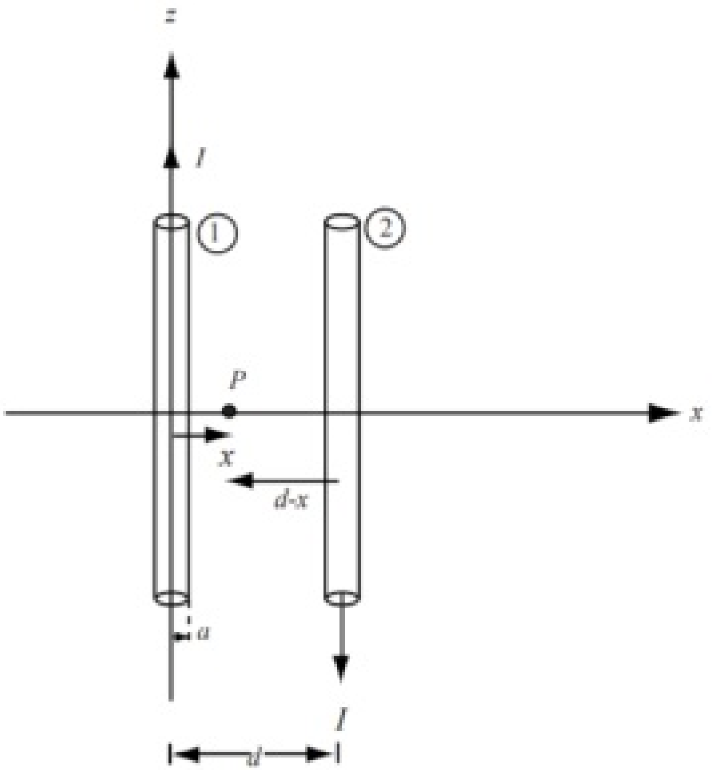
H=4[2μoI(sin45o+sin45o)/ 4πL]

=2root(2) μoI/ πa

Question: Calculate the internal and external inductance per

unit length of a transmission line consisting of two long parallel conducting wires of radius a that carry current in opposite directions the axes of the wires are separated by a distance d, which is much larger than a.

Answer:



Let us place the two wires in the x–z plane and orient the current in one of them to be along the +z-direction and the current in the other one to be along the −z-direction, as shown in Fig. From Eq., the magnetic field at point P = (x,0,z) due to wire 1 is

$$\mathbf{B}_1 = \hat{\boldsymbol{\phi}} \frac{\mu I}{2\pi r} = \hat{\mathbf{y}} \frac{\mu I}{2\pi x},$$

where the permeability has been generalized from free space to any substance with permeability μ, and it has been recognized that in the x-z plane, ˆφ = yˆ and r = x as long as x > 0.

Given that the current in wire 2 is opposite that in wire 1, the magnetic field created by wire 2 at point P = (x,0,z) is in the same direction as that created by wire 1, and it is given by

$$\mathbf{B}_2 = \hat{\mathbf{y}} \frac{\mu I}{2\pi(d-x)}.$$

Therefore, the total magnetic field in the region between the wires is

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = \hat{\mathbf{y}} \frac{\mu I}{2\pi} \left(\frac{1}{x} + \frac{1}{d-x} \right) = \hat{\mathbf{y}} \frac{\mu I d}{2\pi x(d-x)}.$$

From Eq. , the flux crossing the surface area between the wires over a length l of the wire structure is

$$\begin{aligned}
\Phi = \iint_S \mathbf{B} \cdot d\mathbf{s} &= \int_{z=z_0}^{z_0+l} \int_{x=a}^{d-a} \left(\hat{\mathbf{y}} \frac{\mu I d}{2\pi x(d-x)} \right) \cdot (\hat{\mathbf{y}}\, dx\, dz) \\
&= \frac{\mu I l d}{2\pi} \left(\frac{1}{d} \ln\left(\frac{x}{d-x} \right) \right) \Bigg|_{x=a}^{d-a} \\
&= \frac{\mu I l}{2\pi} \left(\ln\left(\frac{d-a}{a} \right) - \ln\left(\frac{a}{d-a} \right) \right) \\
&= \frac{\mu I l}{2\pi} \times 2 \ln\left(\frac{d-a}{a} \right) = \frac{\mu I l}{\pi} \ln\left(\frac{d-a}{a} \right).
\end{aligned}$$

From Eq. , the flux crossing the surface area between the wires over a length l of the wire structure is

$$\Phi = \iint_S \mathbf{B} \cdot d\mathbf{s} = \int_{z=z_0}^{z_0+l} \int_{x=a}^{d-a} \left(\hat{\mathbf{y}} \frac{\mu I d}{2\pi x(d-x)} \right) \cdot (\hat{\mathbf{y}}\, dx\, dz)$$

$$= \frac{\mu I l d}{2\pi} \left(\frac{1}{d} \ln \left(\frac{x}{d-x} \right) \right) \Bigg|_{x=a}^{d-a}$$

$$= \frac{\mu I l}{2\pi} \left(\ln \left(\frac{d-a}{a} \right) - \ln \left(\frac{a}{d-a} \right) \right)$$

$$= \frac{\mu I l}{2\pi} \times 2 \ln \left(\frac{d-a}{a} \right) = \frac{\mu I l}{\pi} \ln \left(\frac{d-a}{a} \right).$$

Since the number of 'turns' in this structure is 1, Eq. states that the flux linkage is the same as magnetic flux: $\Lambda = \Phi$. Then Eq. gives a total inductance over the length l as

$$L = \frac{\Lambda}{I} = \frac{\Phi}{I} = \frac{\mu l}{\pi} \ln \left(\frac{d-a}{a} \right) \quad \text{(H)}.$$

Therefore, the inductance per unit length is

$$L' = \frac{L}{l} = \frac{\mu}{\pi} \ln \left(\frac{d-a}{a} \right) \approx \frac{\mu}{\pi} \ln \left(\frac{d}{a} \right) \quad \text{(H/m)},$$

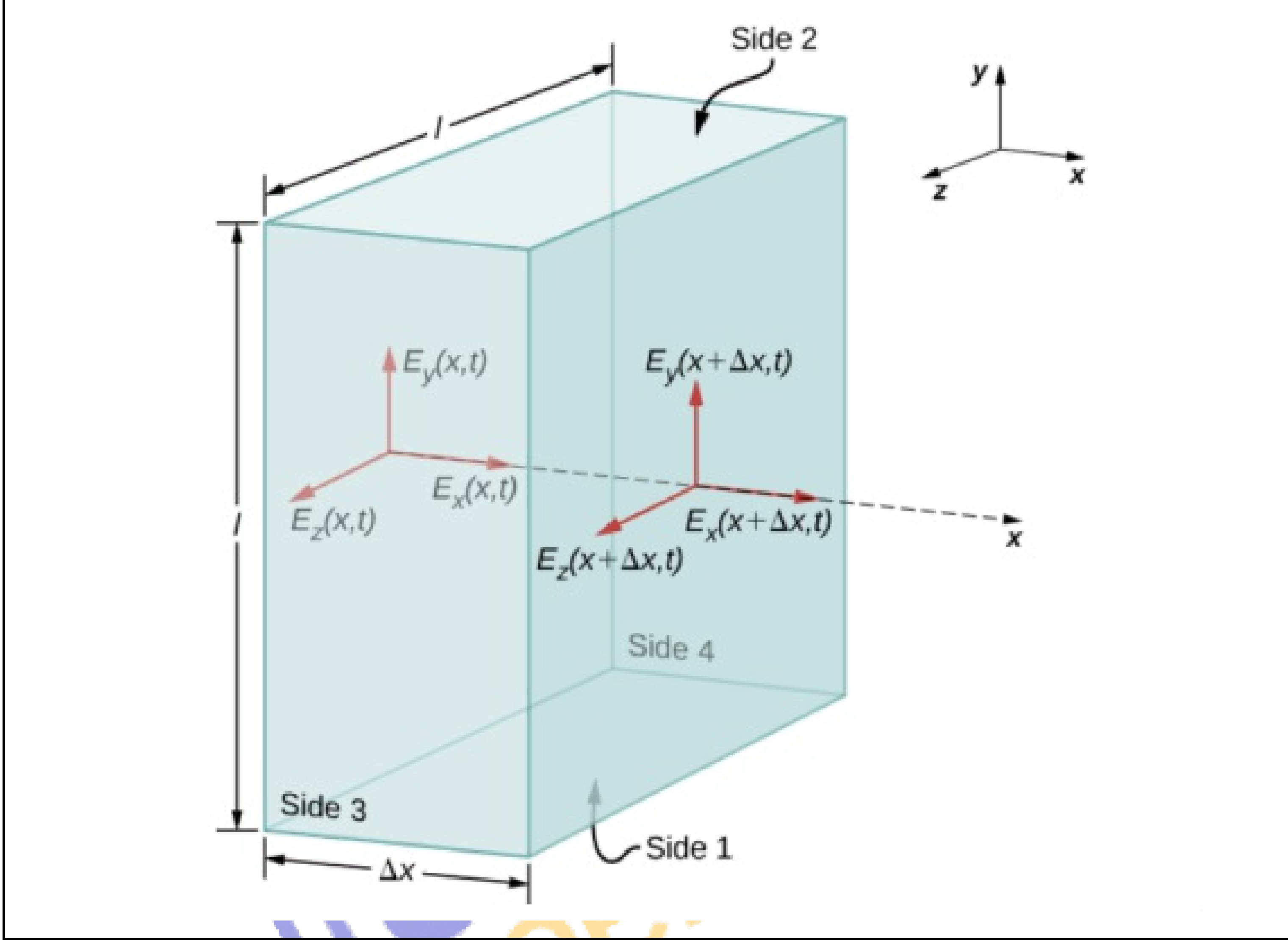
Where the last approximation recognizes that the wires are thin compared to the separation distance (i.e., that $d \gg a$). This has been an implied condition from the beginning of this analysis, where the flux passing through the wires themselves have been ignored.

Question. Show that for a plane electromagnetic wave in free space, the unit vector in the direction of propagation, the electric field vector and magnetic field vector are mutually perpendicular

Assume the Gaussian surface to be the surface of a rectangular box whose cross-section is a square of side l and whose third side has length Δx, as shown in Figure. Because the electric field is a function only of x and t, the y-component of the electric field is the same on both the top (labeled Side 2) and bottom (labeled Side 1) of the box, so that these two contributions to the flux cancel. The corresponding argument also holds for the net flux from the z-component of the electric field through Sides 3 and 4. Any net flux through the surface therefore comes entirely from the x-component of the electric field. Because the electric field has no y- or z-dependence, $E_x(x,t)$ is constant over the face of the box with area A and has a possibly different value $E_x(x+\Delta x,t)$ that is constant over the opposite face of the box. Applying Gauss's law gives

$$\text{Net flux}=-E_x(x,t)A+E_x(x+\Delta x,t)A=Q_{in}\varepsilon_0$$

Where $A=l\times l$ is the area of the front and back faces of the rectangular surface. But the charge enclosed is $Q_{in}=0$, so this component's net flux is also zero, and Equation implies $E_x(x,t)=E_x(x+\Delta x,t)$ for any Δx. Therefore, if there is an x-component of the electric field, it cannot vary with x. A uniform field of that kind would merely be superposed artificially on the traveling wave, for example, by having a pair of parallel-charged plates. Such a component $E_x(x,t)$ would not be part of an electromagnetic wave propagating along the x-axis; so $E_x(x,t)=0$ for this wave. Therefore, the only nonzero components of the electric field are $E_y(x,t)$ and $E_z(x,t)$, perpendicular to the direction of propagation of the wave.



A similar argument holds by substituting E for B and using Gauss's law for magnetism instead of Gauss's law for electric fields. This shows that the B field is also perpendicular to the direction of propagation of the wave. The electromagnetic wave is therefore a transverse wave, with its oscillating electric and magnetic fields perpendicular to its direction of propagation.

Question. What is intrinsic impedance? Drive an expression for it. A plane polarized wave is travelling along z-axis

The intrinsic impedance is a property of a medium - an area of spa
For a vacuum (outer space) or for wave propagation through the
around earth (often called 'free space'), the intrinsic impedance (of
written as η or Z) is given by:

$$\eta = Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi 10^{-7}}{8.854e-12}} \approx 120\pi \approx 377 \text{ Ohms}$$

This parameter is the ratio of the magnitude of the E-field to magnitude of the H-field for a plane wave in a lossless medium (z conductivity):

$$Z = \frac{|\mathbf{E}|}{|\mathbf{H}|}$$

This relation can be derived directly from Maxwell's Equations. Fo general medium with permittivity and permeability given $(\varepsilon, \mu) = (\varepsilon_r \varepsilon_0, \mu_r \mu_0)$, the intrinsic impedance is given by:

$$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0}} = \sqrt{\frac{\mu_r}{\varepsilon_r}} Z_0$$

For a medium with a conductivity σ associated with it, the intrin impedance is given by:

$$Z = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

When the conductivity is non-zero, the above intrinsic impedance i complex number, indicating that the electric and magnetic fields are in-phase.

The intrinsic impedance of free-space has nothing to do with electrical impedance of an antenna. Also, there is no reason to have impedance of an antenna match the intrinsic impedance of free spa (no mismatch loss occurs).

Question. What is the characteristics impedance of transmission line? Derive its expression.

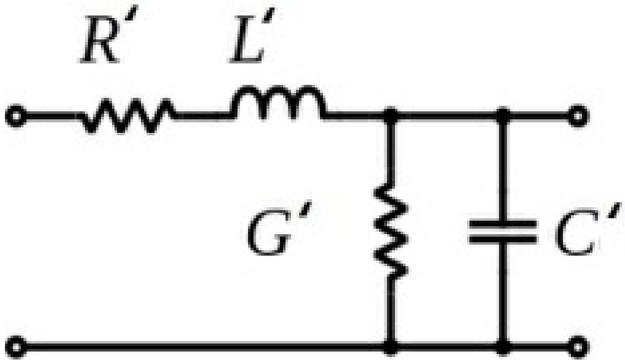
Any media that can support a electromagnetic wave has a characteristic impedance associated with it. Although characteristic impedance units are in Ohms, it is not a "real" impedance you can measure using direct current equipment such as a DC Ohmmeter. And although transmission lines have real loss at microwave frequencies, this isn't what we're talking about either.

The best way to think about characteristic impedance it envision an infinitely long transmission line, which means that there will be no reflections from the load. Placing an alternating current voltage Vin(t) will result in a current Iin(t). The impedance of the transmission line is then:

$$Z_0 = \frac{V_{in}(t)}{I_{in}(t)}$$

Sounds simple enough, but unless you are dealing with "free space", there is no transmission line that is infinitely long. But that equation is starting to look like a version of Ohm's law, where R=V/I.

let's look at the general equivalent circuit of an infinitesimally small piece of a transmission line. All circuits elements are normalized to length in transmission line models; in the metric system the units are Ohms/meter, Farads/meter, mhos/meter and Henries per meter, we will use the "prime" notation when we are discussing quantities that are normalized per unit length.



The T-line model is repeated infinite times along the length of a real transmission line. Hmm, this is starting to sound like calculus, which we have pledged to avoid on Microwaves101. And we will, so we'll stop with this one section. For microwave engineers, the general expression that defines characteristic impedance is:

$$Z_0 = \sqrt{\frac{Z'}{Y'}}$$

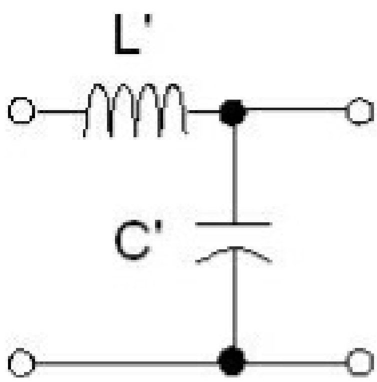
$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

Here R', G', L' and C' are normalized to length, the same as in the model. Note that in its general form, characteristic impedance can be a complex number. Also note that it only becomes complex if either R' or G' are non-zero, which will give you a headache if you think about it too long. In practice we try to achieve nearly lossless transmission lines. For a low-loss transmission line, the following relationships will occur:

$$G' << j\omega C'$$

$$R' << j\omega L'$$

Then for all practical purposes we can ignore the contributions of R' and G' from the equation and end up with a nice scalar quantity for characteristic impedance. For lossless transmission lines the transmission line model reduces to this:



and the more familiar equation for characteristic impedance is simply:

$$Z_0 = \sqrt{\frac{L'}{C'}}\,(ohms)$$

L' is the tendency of a transmission line to oppose a change in current, while C' is the tendency of a transmission line to oppose a change in voltage. Characteristic impedance is a measure of the balance between the two.