

**Aryabhata Knowledge University (AKU)**

**Computer Science and Engineering (CSE)**

**Mathematics-III**

**Solved Exam Paper 2019**

**Q 1 a) If  $y = A \cos(\log x) + B \sin(\log x)$ , Show that  $x^2 y_{n+2} + (2n+1) y_{n+1} + (n^2 + 1) y_n = 0$  where  $y_n = \frac{d^n y}{dx^n}$**

If  $y = A \cos(\log x) + B \sin(\log x)$  -----(1)

Differentiating (1) w.r.t x, we get

$$y_1 = -a \sin(\log x) \frac{1}{x} + b \cos(\log x) \frac{1}{x}$$

$$xy_1 = -a \sin(\log x) + b \cos(\log x) -----(2)$$

Diff 2 again w.r.t x, then we get

$$xy_2 + y_1 = -a \cos(\log x) \frac{1}{x} - b \sin(\log x) \frac{1}{x}$$

$$x^2 y_2 + xy_1 = -[a \cos(\log x) + b \sin(\log x)]$$

$$x^2 y_2 + xy_1 = -y$$

$$y_2 x^2 + y_1 x + y = 0 -----(3)$$

Diff 3 by Leibnitzle theorem n times, we get

$$[y_{n+2}x^2 + nc_1 y_{n+1}2x + nc_2 y_n.2] + [y_{n+1}x + nc_1 y_{n-1}] + y_n = 0$$

$$x^2y_{n+2} + 2nxy_{n+1} + xy_{n+1} + n(n-1)y_n + ny_n + y_n = 0$$

$$x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

**Q2 a) Show that the following function is continuous at the point (0,0):**

$$f(x,y) = \begin{cases} \frac{2x^3 + 3y^3}{x^2 + y^2} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$$

Answer:

$$f(x,y) = \begin{cases} \frac{2x^3 + 3y^3}{x^2 + y^2} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$$

$$0 < = \left| \frac{2x^3 + 3y^3}{x^2 + y^2} \right| < = \frac{2|x^3|}{x^2 + y^2} + \frac{3|y^3|}{x^2 + y^2} = 0$$

$$0 < = \frac{2|x^3|}{x^2} + \frac{3|y^3|}{y^2} = 2|x| + 3|y|$$

$$\lim_{(x,y) \rightarrow (0,0)} 2|x| + 3|y| = 0$$

∴ by the squeeze theorem, we conclude that

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y) = (0,0) \text{ that is } \Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{2x^3 + 3y^3}{x^2 + y^2} = 0$$

∴ f is continuous at (0,0)

**Q2 b) If  $z(x+y) = x^2 + y^2$  show that**

$$\left( \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)^2 = 4 \left( 1 - \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \right)$$

solution:

$$z(x+y) = x^2 + y^2$$

$$z = \frac{x^2 + y^2}{x+y}$$

$$\frac{\partial z}{\partial x} = \frac{(x+y)2x - (x^2 + y^2) \cdot 1}{(x+y)^2} = \frac{x^2 + 2xy - y^2}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{(x+y)2y - (x^2 + y^2) \cdot 1}{(x+y)^2} = \frac{-x^2 + 2xy + y^2}{(x+y)^2}$$

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right) = \frac{x^2 + 2xy - y^2}{(x+y)^2} = \frac{-x^2 + 2xy + y^2}{(x+y)^2} = \frac{2(x-y)}{(x+y)}$$

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = \frac{4(x-y)^2}{(x+y)^2}$$

-----1

$$4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right) = 4\left[1 - \frac{x^2 + 2xy - y^2}{(x+y)^2} - \frac{-x^2 + 2xy + y^2}{(x+y)^2}\right]$$

$$= \frac{4(x-y)^2}{(x+y)^2}$$

-----2

From 1 & 2

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

**Q3 a) Transform the equation**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{into polar coordinates.}$$

Solution:

we have  $x = r \cos \theta$  ,  $y = r \sin \theta$

$$r^2 = x^2 + y^2 \quad , \quad \theta = \tan^{-1} y/x$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x} + \frac{\partial u}{\partial \theta} \cdot \frac{\partial \theta}{\partial x} \Rightarrow \frac{\partial u}{\partial x} (\cos \theta) - \frac{\partial u \sin \theta}{\partial \theta r}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} \right)$$

$$= \left( \cos \theta \frac{\partial}{\partial r} - \frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \right) \left( \cos \theta \frac{\partial u}{\partial r} - \frac{\sin \theta}{r} \frac{\partial u}{\partial \theta} \right)$$

$$\begin{aligned}
&= \cos\theta \frac{\partial}{\partial x} \left( \cos\theta \frac{\partial u}{\partial r} - \frac{\sin\theta}{r} \left( \frac{\partial u}{\partial \theta} \right) \right) - \frac{\sin\theta}{r} \cdot \frac{\partial}{\partial \theta} \left( \cos\theta \frac{\partial u}{\partial r} - \frac{\sin\theta}{r} \right) \\
&= \cos\theta \left( \cos\theta \frac{\partial^2 u}{\partial r^2} + \frac{\sin\theta \partial u}{r^2 \partial \theta} - \frac{\sin\theta}{r} \cdot \frac{\partial^2 u}{\partial r \partial \theta} \right) - \frac{\sin\theta}{r} \left( -\sin\theta \frac{\partial u}{\partial r} + \right. \\
&\quad \left. \cos\theta \frac{\partial^2 u}{\partial r \partial \theta} - \frac{\cos\theta \partial u}{r \partial \theta} - \frac{\sin\theta \partial^2 u}{r \partial \theta^2} \right) \\
&= \cos^2 \theta \frac{\partial^2 u}{\partial r^2} + \frac{\sin\theta \cos\theta \partial u}{r^2 \partial \theta} - \frac{\sin\theta \cos\theta \partial^2 u}{r \partial r \partial \theta} + \frac{\sin^2 \theta \partial u}{r \partial r} - \\
&\quad \frac{\sin\theta \cos\theta}{r} \frac{\partial^2 u}{\partial \theta \partial u} + \frac{\sin\theta \cos\theta \partial u}{r^2 \partial \theta} + \frac{\sin^2 \theta \partial^2 u}{r^2 \partial \theta^2} \\
&= \cos^2 \theta \frac{\partial^2 u}{\partial r^2} + 2 \frac{\sin\theta \cos\theta \partial u}{r^2 \partial \theta} - 2 \frac{\sin\theta \cos\theta \partial^2 u}{r \partial r \partial \theta} + \frac{\sin^2 \theta \partial u}{r \partial r} + \\
&\quad \frac{\sin^2 \theta \partial^2 u}{r^2 \partial \theta^2}
\end{aligned}$$

(1)

$$\begin{aligned}
&= \frac{\partial u}{\partial y} = \frac{\partial u}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial u}{\partial \theta} \frac{\partial \theta}{\partial y} \\
&= \frac{\partial u}{\partial r} y/r + \frac{\partial u}{\partial \theta} \frac{x}{(x^2 + y^2)} \\
&= \frac{\partial u}{\partial r} \sin\theta + \frac{\partial u}{\partial \theta} \frac{\cos\theta}{r} \\
&= \frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right)
\end{aligned}$$

$$= \left( \sin\theta \frac{\partial}{\partial u} + \frac{\cos\theta}{r} \frac{\partial}{\partial \theta} \right) \left( \sin\theta \frac{\partial u}{\partial r} + \frac{\cos\theta \partial u}{r \partial \theta} \right)$$

$$= \sin\theta \frac{\partial}{\partial r} \left( \sin\theta \frac{\partial u}{\partial r} + \frac{\cos\theta \partial u}{r \partial \theta} \right) + \frac{\cos\theta}{r} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial u}{\partial r} + \frac{\cos\theta \partial u}{r \partial \theta} \right)$$

$$\begin{aligned}
&= \sin\theta \left[ \sin\theta \frac{\partial^2 u}{\partial r^2} - \frac{\cos\theta \partial u}{r^2 \partial \theta} + \frac{\cos\theta}{r} + \frac{\partial^2 u}{\partial r \partial \theta} \right] + \frac{\cos\theta}{r} \left[ \cos\theta \left[ \frac{\partial u}{\partial r} + \right. \right. \\
&\quad \left. \left. \sin\theta \frac{\partial^2 u}{\partial r \partial \theta} - \frac{\sin\theta \partial u}{r \partial \theta} + \frac{\cos\theta \partial^2 u}{r \partial \theta^2} \right] \right]
\end{aligned}$$

$$= \sin^2\theta \frac{\partial^2 u}{\partial r^2} - \frac{\sin\theta \cos\theta \partial u}{r^2 \partial \theta} + \frac{\sin\theta \cos\theta \partial^2 u}{r \partial r \partial \theta} + \frac{\cos^2\theta}{r} \left(\frac{\partial u}{\partial r}\right) + \frac{\sin\theta \cos\theta}{r} \left(\frac{\partial^2 u}{\partial r \partial \theta}\right) - \frac{\sin\theta \cos\theta \partial u}{r^2 \partial \theta} + \left(\frac{\partial^2 u}{\partial \theta^2}\right)$$

$$= \sin^2\theta \frac{\partial^2 u}{\partial r^2} - 2 \frac{\sin\theta \cos\theta \partial u}{r^2 \partial \theta} + 2 \frac{\sin\theta \cos\theta \partial^2 u}{r \partial r \partial \theta} + \frac{\cos^2\theta}{r} \left(\frac{\partial u}{\partial r}\right) + \left(\frac{\partial^2 u}{\partial \theta^2}\right)$$


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(2)

By adding 1 & 2

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = (\sin^2\theta + \cos^2\theta) \frac{\partial^2 u}{\partial r^2} + (\sin^2\theta + \cos^2\theta) \frac{1}{r} \frac{\partial u}{\partial r} + (\sin^2\theta + \cos^2\theta) \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2}$$

$$= \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} \right) \quad \text{ans}$$

**Q4 Find the extreme values of  $f(x, y, z) = 2x + 3y + z$ , such that  $x^2 + y^2 = 5$  and  $x + z = 1$**

$$f(x, y, z) = 2x + 3y + z \quad \text{-----(1)}$$

$$f(x, y) = (x^2 + y^2) - 5 \quad \text{-----(2)}$$

$$y(x, z) = x + z - 1 \quad \text{-----(3)}$$

Lagranges Multipliers Equations are

$$\frac{\partial f}{\partial x} + \lambda \frac{\partial \phi}{\partial x} + m \frac{\partial \psi}{\partial x} = 0$$

$$\frac{\partial f}{\partial y} + \lambda \frac{\partial \phi}{\partial y} + m \frac{\partial \psi}{\partial y} = 0$$

$$\frac{\partial f}{\partial z} + \lambda \frac{\partial \phi}{\partial z} + m \frac{\partial \psi}{\partial z} = 0$$

$$2 + \lambda(2x) + m(1) = 0 \quad \text{-----(4)}$$

$$3 + \lambda(2y) + m(0) = 0 \quad \text{-----(5)}$$

$$1 + \lambda(0) + m(1) = 0 \quad \text{-----(6)} \quad \Rightarrow m = -1$$

putting the values of m in (4) and (5), we get

$$2 + 2\lambda x - 1 = 0 \Rightarrow 2\lambda x = -1, \quad x = -\frac{1}{2\lambda}$$

$$3 + 2\lambda y = 0 \Rightarrow 2\lambda y = -3, \quad y = -\frac{3}{2\lambda}$$

putting the values of x,y in  $x^2 + y^2 = 5$ , we get

$$\frac{1}{4\lambda^2} + \frac{9}{4\lambda^2} = 5 \Rightarrow \frac{10}{4\lambda^2} = 5$$

$$2\lambda^2 = 1 \Rightarrow \lambda = \pm \frac{1}{\sqrt{2}}$$

$$\text{we know that, } x = -\frac{1}{2\lambda} = \pm \frac{\sqrt{2}}{2} = \pm \frac{1}{\sqrt{2}}$$

$$y = -\frac{3}{2\lambda} = \pm \frac{3\sqrt{2}}{2} = \pm \frac{3}{\sqrt{2}}$$

From (3),  $x+z=1$  or  $z=1-x$

$$z = 1 \pm \frac{1}{\sqrt{2}}$$

putting  $x = \frac{1}{\sqrt{2}}$ ,  $y = \frac{3}{\sqrt{2}}$  and  $z = 1 - \frac{1}{\sqrt{2}}$  in equation (1), we get

$$f = \frac{2}{\sqrt{2}} + \frac{9}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} = \frac{10}{\sqrt{2}} + 1 = 5\sqrt{2} + 1$$

putting  $x = -\frac{1}{\sqrt{2}}$ ,  $y = -\frac{3}{\sqrt{2}}$  and  $z = 1 + \frac{1}{\sqrt{2}}$  in equation (1), we get

$$f = 2\frac{1}{\sqrt{2}} + 3\left(\frac{-3}{\sqrt{2}}\right) + \left(1 + \frac{1}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}} - \frac{9}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}}$$

**Q5 Evaluate  $\oiint_s F \cdot n \cdot ds$  where  $F = 4xz \hat{i} + y^2 \hat{j} + yzk \hat{k}$  a surface of the cube bounded by  $x=0, y=1, z=0, x=1, y=0, z=1$  using Gauss divergence theorem**

s.no	surface	$n$	$ds$	axis
1	OABC	$-\hat{k}$	$dx dy$	$z$
2	DEFG	$\hat{k}$	$dx dy$	$z$
3	OAFG	$-\hat{j}$	$dx dz$	$y$
4	BCDE	$\hat{j}$	$dx dz$	$y$
5	ABEF	$\hat{i}$	$dy dz$	$x$
6	OCDG	$-\hat{i}$	$dy dz$	$x$

$$\oiint_s F \cdot n \cdot ds = \oiint_{OABC} F \cdot n \cdot ds + \oiint_{DEFG} F \cdot n \cdot ds + \oiint_{OAFG} F \cdot n \cdot ds + \oiint_{BCDE} F \cdot n \cdot ds + \oiint_{ABEF} F \cdot n \cdot ds + \oiint_{OCDG} F \cdot n \cdot ds$$



$$\iint_{BCDE} \mathbf{F} \cdot \hat{n} \, ds + \iint_{ABEF} \mathbf{F} \cdot \hat{n} \, ds +$$

$$\iint_{OCDG} \mathbf{F} \cdot \hat{n} \, ds \quad \text{-----} ($$

$$\iint_{OABC} \mathbf{F} \cdot \hat{n} \, ds = \iint_{OABC} (4xz \hat{i} + y^2 \hat{j} + yz \hat{k}) \cdot (-\hat{k}) \, dx \, dy$$

$$= \int_0^1 \int_0^1 -yz \, dx \, dy = 0 \quad (\text{as } z = 0)$$

$$\iint_{DEFG} (4xz \hat{i} + y^2 \hat{j} + yz \hat{k}) \cdot \hat{k} \, dx \, dy = \iint_{DEFG} yz \, dx \, dy$$

$$\int_0^1 \int_0^1 y(1) \, dx \, dy = \int_0^1 dx \left[ \frac{y^2}{2} \right]_{0-1} = [x]_{0-1} = 1/2$$

$$\iint_{OAFG} (4xz \hat{i} + y^2 \hat{j} + yz \hat{k}) \cdot (-\hat{j}) \, dx \, dz = \iint_{OAFG} -y^2 \, dx \, dz = 6 \quad (\text{as } y = 0)$$

$$\iint_{BCDE} (4xz \hat{i} + y^2 \hat{j} + yz \hat{k}) \cdot \hat{j} \, dx \, dz = \iint_{BCDE} y^2 \, dx \, dz$$

$$= \int_0^1 dx \int_0^1 dz = (x)_{0-1} (z)_{0-1} = -1 \quad (\text{as } y = 1)$$

$$\iint_{ABEF} (4xz \hat{i} + y^2 \hat{j} + yz \hat{k}) \cdot \hat{i} \, dy \, dz = \iint_{ABEF} 4xz \, dy \, dz$$

$$= \int_0^1 \int_0^1 4(1)z \, dy \, dz \Rightarrow 4(y)_{0-1} \left( \frac{z^2}{2} \right)_{0-1} = 4(1) \left( \frac{1}{2} \right) = 2$$

$$\iint_{OCDG} (4xz \hat{i} + y^2 \hat{j} + yz \hat{k}) \cdot (-\hat{i}) \, dy \, dz = \int_0^1 \int_0^1 -4(1)z \, dy \, dz = 0 \quad (\text{as } x =$$

on putting these values in (1), we get

$$\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = 0 + 1/2 + 6 - 1 + 2 + 0 \Rightarrow 3/2$$

Q 6 (a) Evaluate  $\frac{\partial}{\partial \theta} \{ \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) \}$

$$(\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos \theta & -\sin \theta & -3 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= \hat{i} (\sin \theta + 9) - \hat{j} (-\cos \theta + 6) + \hat{k} (3 \cos \theta + 2 \sin \theta)$$

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \sin \theta & \cos \theta & \theta \\ (\sin \theta + 9) & (\cos \theta - 6) & (3 \cos \theta + 2 \sin \theta) \end{vmatrix}$$

$$= \hat{i} [\cos \theta (3 \cos \theta + 2 \sin \theta) - \theta (\cos \theta - 6)]$$

$$- \hat{j} [\sin \theta (3 \cos \theta + 2 \sin \theta) - \theta (\sin \theta + 9)]$$

$$+ \hat{k} [\sin \theta (\cos \theta - 6) - \cos \theta (\sin \theta + 9)]$$

at  $\theta = 0$

$$\Rightarrow \hat{i} [\cos 0 (3 \cos 0 + 2 \sin 0) - 0 (\cos 0 - 6)]$$

$$- \hat{j} [\sin 0 (3 \cos 0 + 2 \sin 0) - 0 (\sin 0 + 9)]$$

$$- \hat{k} [\sin 0 (\cos 0 - 6) - \cos 0 (\sin 0 + 9)]$$

$$\Rightarrow 3\hat{i} - 9\hat{k} \quad \text{ans}$$

**Q 6 A particle moves along the curve  $x=t^3 + 1$ ,  $y=t^2$ ,  $z= 2t+5$  where  $t$  is the time. Find the components of the velocity and acceleration at  $t=1$  in the direction  $\hat{i}+\hat{j}+3\hat{k}$**

$$x = t^3 + 1, y = t^2, z = 2t + 5$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = (t^3 + 1)\hat{i} + (t^2)\hat{j} + (2t + 5)\hat{k}$$

$$\text{velocity} = \frac{dr}{dt} = 3t^2 \hat{i} + 2t\hat{j} + 2\hat{k}$$

$$\text{when } t = 1, \text{ we have, } \frac{dr}{dt} = 3\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\text{unit velocity along } (\hat{i} + \hat{j} + 3\hat{k}) = (\hat{i} + \hat{j} + 3\hat{k}) / \sqrt{1 + 1 + 9}$$

$$= \frac{1}{\sqrt{11}}(\hat{i} + \hat{j} + 3\hat{k})$$

$$\text{component of velocity } (3\hat{i} + 2\hat{j} + 2\hat{k}) \text{ along } (\hat{i} + \hat{j} + 3\hat{k})$$

$$= (3\hat{i} + 2\hat{j} + 2\hat{k}) \cdot \frac{1}{\sqrt{11}}(\hat{i} + \hat{j} + 3\hat{k})$$

$$= \frac{1}{\sqrt{11}}(3+2+6) = \frac{11}{\sqrt{11}} = \sqrt{11} \quad \text{ans}$$

## Q7 (a) Solve by the method of variation of parameter

$$\frac{d^2y}{dx^2} + n^2y = \sec nx$$

solution: -  $(D^2 + n^2)y = \sec nx$

$$m^2 + n^2 = 0 \Rightarrow m = \pm ni$$

$$cf = c_1 \cos nx + c_2 \sin nx$$

$$y = A' \cos nx + B' \sin nx \quad \text{-----(I)}$$

by diff. Equation (1)

$$(-nA' \sin nx + B' n \cos nx = \sec nx) \quad \text{-----(II)}$$

Now multiplying by  $n \sin nx$  in equation (I) & multiplying by equation (II)

$$n \sin nx (A' \cos nx + B' \sin nx = 0)$$

$$\cos nx (-nA' \sin nx + B' n \cos nx = \sec nx)$$

$$A' n \sin nx \cos nx + B' n \sin^2 nx = 0$$

$$-A' n \sin nx \cos nx + B' n \cos^2 nx = 1$$

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$$nB' (\sin^2 nx + \cos^2 nx) = 1$$

$$B' = \frac{1}{n}$$

$$\frac{dB}{dx} = \frac{1}{n}$$

$$dB = \frac{dx}{n}$$

$$B = \frac{x}{n} + C$$

$$A' \cos nx + B' \sin nx = 0$$

$$A' \cos nx + \frac{1}{n} \sin nx = 0$$

$$A' = -\frac{1 \sin nx}{n \cos nx} = -\frac{1}{n} \tan nx$$

$$\int dA = -\frac{1}{n} \int \tan nx \, dx + C_2$$

$$A = -\frac{1}{n^2} \log \sec nx + C_2$$

$$y = \left[ -\frac{1}{n^2} \log \sec nx + C_2 \right] \cos nx + \left( \frac{x}{n} + C_1 \right) \sin nx$$

$$Y = C_1 \sin nx + C_2 \cos nx + \frac{x}{n} \sin nx - \frac{1}{n^2} \cos nx \cdot \log \sec nx$$

**Q 7 (b) solve  $\frac{d^2y}{dx^2} - 2 \tan x \frac{dy}{dx} + 5y = \sec x \cdot e^x$**

$$\frac{d^2y}{dx^2} - 2 \tan x \frac{dy}{dx} + 5y = \sin x \cdot e^x$$

$$y'' - 2 \tan x y' + 5y = \sin x \cdot e^x$$

$$\text{compare with } y'' + 8y' + qy = R$$

$$p = -2 \tan x, \quad q = 6, \quad R = \sin x \cdot e^x$$

$$\text{for C.F} \quad \mu = e^{-1/2 \int p dx}$$

$$q_1 = q - \frac{1}{2} \frac{dp}{dx} - \frac{p^2}{4}$$

$$R_1 = \frac{R}{\mu}$$

$$\mu = e^{-1/2 \int -2 \tan x dx} = e^{\log \sec x} = \sec x$$

$$q_1 = 5 - \frac{d}{dx}(-2 \tan x) - \frac{4 \tan^2 x}{4}$$
$$= 5 + \sec^2 x - \tan^2 x = 6$$

$$R_1 = \frac{\sin x e^x}{4}$$

$$\frac{d^2v}{dx^2} + q_1 v = R_1$$

$$(D^2 + 6)v = \frac{\sin x e^x}{4}$$

$$\text{A.E} = m^2 + 6 = 0 \Rightarrow m^2 = -6 \Rightarrow m = \pm 6i$$

$$\text{CF} = (C_1 \cos 6x + C_2 \sin 6x)$$

$$\text{P.I} = \frac{1}{(D^2 + 6)} \frac{\sin x e^x}{4} \Rightarrow \frac{1}{4(D^2 + 6)} (\sin x \cdot e^x)$$

$$= \frac{e^x}{4[(D+1)^2 + 6]} \sin x$$

$$= \frac{e^x}{4(D^2 + 1 + 2D + 6)} \sin x \Rightarrow \frac{e^x}{4(-1^2 + 1 + 2D + 6)} \sin x$$

$$= \frac{e^x}{4(-1+1+2D+6)} \sin x \Rightarrow \frac{e^x}{4(2D+6)} \sin x$$

$$= \frac{e^x D-3}{8(D^2-9)} \sin x \Rightarrow \frac{e^x D-3}{8(-1-9)} \sin x$$

$$= \frac{e^x}{-80} (D-3) \sin x \Rightarrow \frac{e^x}{-80} (D \sin x - 3 \sin x)$$

$$= \frac{e^x}{-80} (\cos x - 3 \sin x)$$

$$C.S = (C_1 \cos 6x + C_2 \sin 6x) - \frac{e^x}{80} (\cos x - 3 \sin x)$$



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