Aryabhatta Knowledge University (AKU)

Electronics and electrical engineering (ECE)

Mathematics-III

Solved Exam Paper 2019

Q (1) A) what is the Wavelet transform?

Wavelet transform:--

In mathematics, a wavelet series is a representation of a square – integrable function by a certain ortho normal series generated by a wavelet.

A function y Î L^2 is called an orthonormal wavelet if it can be used to define a Hilbert basis that is a complete orthonormal system. For the Hilbert space L^2 (R) of square integrable function.

The Hilbert basis is constructed as the family of Function

{y_{jk} : j, k Î z }

$y_{jk}(x) = 2^{j/2}\ y(2^j x - k)$

for integers j,k Î z

If under the standard inner product on L^2(R) (f,g) = $\int_{-\infty}^{+\infty} f(x)g(x)dx$

this family is orthonormal

Completeness is satisfied if every function

f ÎL^2(R) may be expanded in the basis of

f(x) = $\sum_{j,k=-\infty}^{+\infty}$, C_{jk} y_{jk} (x)

with convergence of the series understood to be convergence in norm.

Such a representation of f is known as a wavelet series.

The integral wavelet transform is defined as

[W_yf] (a,b) = 1/Ö|a| $\int_{-\infty}^{+\infty} \psi(\Gamma - b)/a f(x)dx$

The wavelet coefficient $C_{j,k}$ are then given by

$C_{j,k}$ = [W_yf] (2^{-j} , k2^{-j})

Here, a = 2^{-j} is called the binary dilation

& b = k2^{-j} is the binary position

Q (2) B) For any three sets A, B, C prove that A x (B (AxB)U(AxC)

Answer: To prove : A * (B U C) = (A * B) U (A * C)

Proof :

Let (a, b) Î A * (B U C)

=> a Î A and b Î (B U C)

=> a ÎA and { b Î B or b Î C}

=> {a Î A and b Î B } or {a Î A and b Î C }

=> (a,b) Î (A * B) U (A * C)

\ (a, b) Î A * (B U C) => (a,b) Î (A * B) U (A * C)

\ A * (B U C) Í (A * B) U (A * C)   -------------------------(1)

Again , Let (x,y) Î(A * B) U (A * C)

=> (x,y) Î(A * B) or (x,y) Î(A * C)

=> {x Î A and y Î B} or {x Î A and y Î C}

=>   x Î A and y Î (B U C)

=> (x,y) Î A * (B U C)

\ (x,y) Î(A * B) U (A * C) = (x,y) Î A * (B U C)

\ (A * B) U (A * C) Í A * (B U C)   -----------------------------(2)

Now from equation (1) & (2) we get,

Hence,

A * (B U C) = (A * B) U (A * C)

Hence Proved.

Q (3) A)Discuss Skewness and Kurtosis for the fo frequency distribution:

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
No. of Students	**5**	**10**	**40**	**20**	**25**

Answer: skewness of kurtosis :

Step 1: - To find mean `x

class	x	f	F_x
0-10	5	5	25
10-20	15	10	150
20-30	25	40	1000
30-40	35	20	700

40-50	45	25	1125
		N = å f = 100	å fx = 3(

\ $\bar{x} = \frac{\Sigma fx}{N} = \frac{3000}{100}$

`x = 30

Step 2 : - To find central Moments :

x	f	f(x - `x)	f(x - `x)^2	f(x - `x)^3	f(x
5	5	-125	3125	-78125	1!
15	10	-150	2250	-33750	5(
25	40	-200	1000	-5000	2!
35	20	100	500	-2500	1:
45	25	375	5625	84375	1:
	å f = N = 100	=åf(x - `x) = 0	=åf(x - `x)^2 = 12500	åf(x - `x)^3 = -30000	=åf(: 37(

$m_1 = \frac{\Sigma f(x - \bar{x})}{N} = \frac{0}{100} = 0$

$m_2 = \frac{\Sigma f(x - \bar{x})^2}{N} = \frac{12500}{100} = 125$

$m_3 = \frac{\Sigma f(x - \bar{x})^3}{N} = \frac{-30000}{100} = -300$

$m_4 = \frac{\Sigma f(x - \bar{x})^4}{N} = \frac{-30000}{100} = 37625$

skewness b1 $= \frac{(\mu 3)^2}{(\mu 2)^3} = \frac{(-300)^2}{(125)^3} = 0.046$

$b_1 \neq 0$ Hence distribution is unsymmetrical

$b_2 = \frac{(\mu 4)}{(\mu 2)^2} = \frac{37625}{(125)^2}$

= **2.408** < 3

\ $b_2 < 3$ => Given distribution is Platy Kurtosis.

Q (3) B) In a partially destroyed laboratory record analysis of a correlation data, the following results a eligible:

Variance of x=9

Regression equations:

40x-18y= 214, 8x-10y+66=0

Find i) Mean values of x and, ii) the standard deviat and coefficient of correlation between x and y and iii between the lines of regressions

Answer: Given Regression Equations are

40x-18y = 214 -------- (1) & 8x – 10y +66 = 0 --------- (2)

(i) To find mean values of x and y

Put x = `x & y = `y in given regression equations.

\ 8 `x – 10`y = -66

40`x – 18`y = 214

by solving we get , `x = 13 & `y = 17

(ii) To find correlation

r = Ö (bxy – byx)

solve equation (2) for x & take of y as bxy

From equation (2) => 8x = 10y-66

$x = \frac{10}{8}y - \frac{66}{8}$

\ bxy = $\frac{10}{8}$

Solve equation (1) for y & take coefficient of x as byx

$-18y = 214 - 40x$

$y = \frac{40}{18}x - \frac{214}{18}$

\ byx = $\frac{40}{18}$

[check : bxy . Byx <= 1

If not then swap x <--> y & Num <--> Dero

bxy.byx = $(\frac{10}{8})(\frac{40}{18})$ = 2.77 > 1

\ swap x <--> y & Num <--> Dero

byx = $\frac{8}{10}$& bxy = $\frac{18}{40}$]

\ ¡ = Ö (bxy. byx) = Ö $((\frac{18}{40})(\frac{8}{10}))$ = **0.6**

(iii) Var (x) = 9 & var(y) = ?

\ byx = $\frac{cov.(x,y)}{var(x)}$

$\frac{8}{10} = \frac{cov.(x,y)}{9}$

cov.(x,y) = $\frac{72}{10}$

\ bxy = $\frac{cov.(x,y)}{var(y)}$

$\frac{18}{40}$= $\frac{72}{10}$/var(y)

\ var(y) = 16

s $_{y}$ = std. Deviation of y = Övar(y)

= Ö16

s $_{y}$ = 4

s $_{x}$ = Övar(x) =Ö9 = 3

\ s $_{x}$ = 3

(iii) Angle between the lines of regression

tanq = $\left(\frac{1-r^2}{r}\right)\left(\frac{\sigma x - \sigma y}{\sigma x^2 + \sigma y^2}\right)$

$= \left(\frac{1-(0.6)^2}{0.6}\right)\left(\frac{3.4}{9+16}\right)$

$= \left(\frac{1-0.36}{0.6}\right)\left(\frac{12}{25}\right)$

tanq = 0.512

\ q = 0.47⁰

Q (4) A) Find the mean and variance of b distribution.

Answer: (A) Mean of Binomial Distribution

Mean = E(x) = $\sum_{x=0}^{n} x.p(x)$

$= \sum_{x=0}^{n} x$ ncx. $P^x .q^{n-x}$

$= \sum_{x=0}^{n} x \frac{n!}{(n-x)!x!} P^x .q^{n-x}$

$= \sum_{x=0}^{n} x \frac{n!}{(n-x)!x(x-1)!} P^x .q^{n-x}$

$= \sum_{x=0}^{n} x \frac{n(n-1)!}{[(n-1)-(x-1)]!(x-1)! * q[(n-1)-(x-1)]} (p.p^{x-1})$

$= np \sum_{x=0}^{n} x \frac{n(n-1)!}{[(n-1)-(x-1)]!(x-1)! * q[(n-1)-(x-1)]} p^{x-1}$

$= np\,(p+q)^{n-1}$

$=>$ $(p+q = 1)$

$= np\,(1)^{n-1}$

Mean = np

(B) Variance of Binomial Distribution

Variance $sx^2 = E(x^2) – [E(x)]^2$

\ $E(x^2) = \sum_{x=0}^{n} x^2 p(x)$

$= \sum_{x=0}^{n} [x + (x-1)x]p(x)$

$= \sum_{x=0}^{n} xp(x) + \sum_{x=0}^{n} (x-1)xp(x)$

$= np + \sum x(x-1) ncx\ P^x .q^{n-x}$

$= np + \sum x(x-1)\frac{n!}{(n-x)!x!}P^x .q^{n-x}$

$= np + \sum x(x-1)\frac{n!}{(n-x)!x(x-1)!} P^x .q^{n-x}$

$$= np + \sum\left(\frac{n(n-1)(n-2)!}{[(n-2)!x(x-2)]!(x-2)!}\right)p^2 \cdot p^{x-2} * q^{[(n-2)-(x-2)]}$$

$$= np + n(n-1)p^2 \sum\left(\frac{(n-2)! * p^{(x-2)} * q^{([(n-2)-(x-2)])}}{[(n-2)-(x-2)]!(x-2)!}\right)$$

$$= np + n(n-1)p^2 (p+q)^2$$

$\Rightarrow \quad (p+q = 1)$

$$= np + n^2p^2 - np^2 (1)^{n-2}$$

$$E(x^2) = np + n^2p^2 - np^2$$

$$\text{\variance } (sx^2) = E(x^2) - [E(x)]^2$$

$$= np + n^2p^2 - np^2 - (np)^2$$

$$= np + n^2p^2 - np^2 - n^2p^2$$

$$= np(1-p) \qquad \Rightarrow (q=1-p)$$

Variance = npq

Q (4) B) The probability of pen manufactured by a co

will be defective is 1/10. If 12 such pens are manufa
Find the probability that i) exactly two will be defec
at least two will be defective and iii) none will be effe

Answer: Total number of pens (n) = 12

Probability of a defective pen $p = \frac{1}{10} = 0.1$

Probability of a non - defective pen $q = 1 – p = 0.9$

[=> (p + q = 1)]

(a) Probability that exactly two will be defective

$p(r) = ncr\ p^r q^{n-r}$

\ $p(2) = 12c2\ (0.1)^2(0.9)^{12-2}$

$= \frac{12!}{2!10!}(0.1)^2(0.9)^{10}$

\ p(2) = 0.2301

(b) Probability that none will be defective

$p(0) = 12c0\ (0.1)^0\ (0.9)^{12}$

$= 1*1*\ (0.9)^1$

p(0) = 0.2824

(c)Probability that atleast two will be defective p(2)+p(3)+p(4)+.......+p(12) = 1 – [p(0) + p(1)]

$= 1 - [\ 0.2824 + 12c1\ (0.1)^1\ (0.9)^{11}\ ]$

= 1 - [0.2824 + 0.3766]

= 0.3410

Q (5) A) In a test of 2000 electric bulbs it was found that the life of a particular make was normally distributed with an average life of 2040 hours and SD of 60 hours. Estimate the number of bulbs likely to burn for i) more than 2150 hours, ii) less than 1950 hours, and iii) more than 1950 hours and less than 2160 hours.

Answer: Given m = 2040 & s = 60

(i) For x = 2150

$z = \frac{x-\mu}{\sigma} = \frac{2150-2040}{60}$

= 1.83

Area against z = 1.83 from Table = 0.4664

Required area = 0.5 – 0.4664

= 0.0336

\ The no. Of bulbs likely to burn for more than 2150 has

= 0.0336 * 2000 = 67.2

» 67 bulbs

(ii) Less than 1950

for x = 1950

z = $\frac{x-\mu}{\sigma}$= $\frac{1950-2040}{60}$= - 1.5

Area against z = - 1.5 from Table = 0.4332

Required area = 0.5 – 0.4332

= 0.668

\ The no. Of bulbs likely to burn for less than 1950 has

= 0.668 * 2000

= 153.6

»153 bulbs

(iii) when x = 1950

z = $\frac{1950-2040}{60}$

= - 1.5

when x = 2160

z = $\frac{2160-2040}{60}$ = 2

p(-1.5<= z <= 2)

p(>= 1.5) = Required Area

= 0.4332

& p(z<=2) = Required Area

= 0.4772

\ p(-1.5<= z <= 2) = 0.4332 + 0.4772

= 0.9104

\ The no. Of bulbs likely to burn for more than 1950 hours but less than 2160 hours

= 0.9104 * 2000 = 1820.8

» 1820 bulbs

Q(5) B) Find the curve of best fit of the type y=a following data by method of least square:

x	**1**	**5**	**7**	**9**	**12**
y	**10**	**15**	**12**	**15**	**21**

Answer: Given curve $y = ae^{bx}$

take log on both sides

$\log y = \log a + \log e^{bx}$

\ $\log y - \log a + bx \log e$

\ $Y = A + B x$

$\Sigma Y = nA + B \Sigma x$ ----------(1)

x	y	Y = log y	x^2	xY
1	10	1	1	1
5	15	1.18	25	5.9
7	12	1.08	49	7.5
9	15	1.18	81	10.
12	21	1.32	144	15.
$\Sigma x = 34$		$\Sigma Y = 5.76$	$\Sigma x^2 = 300$	ΣxY

$\Sigma xY = A \Sigma x + B \Sigma x^2$ -------------(2)

put in Equation (1) & (2)

\ $5.76 = 5A + 34(B)$

40.92 = 34A + 300(B)

by Solving we get,

A = 0.98

B = 0.025

\ log a = A

\ a = $10^{0.98}$ = 9.55

loge(b) = B => b = 1.06

y = ae^{bx}

y = 9.5 $e^{1.06x}$